Continuity

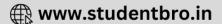**Basic Level**

1. If the function $f(x) = \begin{cases} 5x - 4 & , \text{if } 0 < x \leq 1 \\ 4x^2 + 3bx, & \text{if } 1 < x < 2 \end{cases}$ is continuous at every point of its domain, then the value of b is

   (a) –1   (b) 0   (c) 1   (d) None of these

2. If $f(x) = \begin{cases} \dfrac{\log(1 + 2ax) - \log(1 - bx)}{x}, & x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at $x = 0$, then k equals

   (a) $2a + b$ (b) $2a - b$ (c) $b - 2a$ (d) $b + a$

3. If $f(x) = \begin{cases} x & , \text{ when } 0 \leq x < 1 \\ k - 2x & , \text{ when } 1 \leq x \leq 2 \end{cases}$ is continuous at $x = 1$, then value of k is

   (a) 1   (b) –1   (c) 3   (d) 2

4. If $f(x) = \begin{cases} x & , \; x < 0 \\ 1 & , \; x = 0 \\ x^2 & , \; x > 0 \end{cases}$, then true statement is

   (a) $\lim_{x \to 0} f(x) = 1$ (b) $\lim_{x \to 0} f(x) = 0$ (c) $f(x)$ is continuous at $x = 0$ (d) $\lim_{x \to 0} f(x)$ does not exist

5. If $f(x) = \dfrac{x - a}{\sqrt{x} - \sqrt{a}}$ is continuous at $x = a$, then $f(a)$ equals

   (a) $\sqrt{a}$ (b) $2\sqrt{a}$ (c) a (d) $2a$

6. If $f(x) = |x - b|$, then function

   (a) Is continuous $\forall x$ (b) Is continuous at $x = \infty$ (c) Is discontinuous at $x = b$ (d) None of these

7. If $f(x) = \begin{cases} \dfrac{x^4 - 16}{x - 2}, & \text{when } x \neq 2 \\ 16 & , \text{when } x = 2 \end{cases}$ then

   (a) $f(x)$ is continuous at $x = 2$ (b) $f(x)$ is discontinuous at $x = 2$

   (c) $\lim_{x \to 2} f(x) = 16$ (d) None of these

8. In the following discontinuous function is

   (a) $\sin x$ (b) x^2 (c) $\dfrac{1}{1 - 2x}$ (d) $\dfrac{1}{1 + x^2}$

9. If $f(x) = \begin{cases} x^2 & , \text{when } x \leq 1 \\ x + 5, & \text{when } x > 1 \end{cases}$ then

(a) $f(x)$ is continuous at $x = 1$ (b) $f(x)$ is discontinuous at $x = 1$

(c) $\lim\limits_{x \to 1} f(x) = 1$ (d) None of these

10. If $f(x) = \begin{cases} 1 + x, \text{ when } x \leq 2 \\ 5 - x, \text{ when } x > 2 \end{cases}$ then

(a) $f(x)$ is continuous at $x = 2$ (b) $f(x)$ is discontinuous at $x = 2$ (c) $f(x)$ is discontinuous at $x = 0$ (d) None of these

11. The point of discontinuity of the function $f(x) = \dfrac{1 + \cos 5x}{1 - \cos 4x}$ is

(a) $x = 0$ (b) $x = \pi$ (c) $x = \pi / 2$ (d) All of these

12. Function $f(x) = | x |$ is **[Rajasthan PET 1992]**

(a) Discontinuous at $x = 0$ (b) Discontinuous at $x = 1$ (c) Continuous at all points (d) Discontinuous at all points

13. If $f(x) = \begin{cases} x^2 , & \text{when } x \neq 1 \\ 2 , & \text{when } x = 1 \end{cases}$ then

(a) $\lim\limits_{x \to 1} f(x) = 2$ (b) $f(x)$ is continuous at $x = 1$ (c) $f(x)$ is discontinuous at $x = 1$ (d) None of these

14. Let $f(x) = \begin{cases} \dfrac{\sin \pi x}{5x} , & x \neq 0 \\ k , & x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, then $k =$

(a) $\dfrac{\pi}{5}$ (b) $\dfrac{5}{\pi}$ (c) 1 (d) 0

15. Function $f(x) = x - | x |$ is

(a) Discontinuous at $x = 0$ (b) Discontinuous at $x = 1$ (c) Continuous at all points (d) Discontinuous at all points

16. Function $f(x) = x + | x |$ is

(a) Continuous at all points (b) Discontinuous at $x = 0$ (c) Discontinuous at $x = 1$ (d) Discontinuous at all points

17. If $f(x)$ is continuous function and $g(x)$ is discontinuous function, then correct statement is

(a) $f(x) + g(x)$ is continuous function (b) $f(x) - g(x)$ is continuous function

(c) $f(x) + g(x)$ is discontinuous function (d) $f(x).g(x)$ is discontinuous function

18. Function $f(x) = \begin{cases} -1 , \text{ when } x < -1 \\ -x , \text{ when } -1 \leq x \leq 1 \text{ is continuous} \\ 1 , \text{ when } x > 1 \end{cases}$ **[Rajasthan PET 1986]**

(a) Only at $x = 1$ (b) Only at $x = -1$ (c) At both $x = 1$ and $x = -1$ (d) Neither at $x = 1$ nor at $x = -1$

Advance Level

19. Let $f(x) = \dfrac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$ the value which should be assigned to f at $x = 0$ so that it is continuous everywhere is

[MP PET 1992]

(a) $\dfrac{1}{2}$ (b) -2 (c) 2 (d) 1

20. The value of $f(0)$ so that the function $f(x) = \dfrac{\sqrt{1 + x} - (1 + x)^{1/3}}{x}$ becomes continuous is equal to

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{4}$ (c) 2   (d) $\dfrac{1}{3}$

21. If $f(x) = \begin{cases} \dfrac{|x-a|}{x-a} & \text{when } x \neq a \\ 1 & \text{when } x = a \end{cases}$ then   **[AI CBSE 1983]**

(a) $f(x)$ is continuous at $x=a$ (b) $f(x)$ is discontinuous at $x=a$ (c) $\lim\limits_{x \to a} f(x) = 1$ (d)   None of these

22. If $f(x) = \begin{cases} \dfrac{x}{e^{1/x}+1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ then   **[BIT Rnchi 1999]**

(a) $\lim\limits_{x \to 0^+} f(x) = 1$ (b) $\lim\limits_{x \to 0^-} f(x) = 1$ (c) $f(x)$ is continuous at $x = 0$ (d) None of these

23. If the function $f(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x}, & \text{when } x \neq \dfrac{\pi}{2} \\ 3, & \text{when } x = \dfrac{\pi}{2} \end{cases}$ be continuous at $x = \dfrac{\pi}{2}$, then $k=$

(a) 3   (b) 6   (c) 12   (d) None of these

24. A function $f(x)$ is defined in $[0,1]$ as follows $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$, then correct statement is

(a) $f(x)$ is continuous at $x = 0$ (b) $f(x)$ is continuous at $x = 1$

(c) $f(x)$ is continuous at $x = \dfrac{1}{2}$ (d) $f(x)$ is everywhere discontinuous

25. If $f(x) = \begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, then at $x = 0, f(x)$ is   **[BITS (Mesra) 1998]**

(a) Continuous   (b) Left continuous   (c) Right continuous   (d) None of these

26. The function $f(x) = \begin{cases} x+2, & 1 \leq x \leq 2 \\ 4, & x = 2 \\ 3x-2, & x > 2 \end{cases}$ is continuous   **[DCE 1999]**

(a) $x = 2$ only   (b) $x \leq 2$ (c) $1 \leq x$ (d) None of these

27. If $f(x) = \begin{cases} 1, & \text{when } 0 < x \leq \dfrac{3\pi}{4} \\ 2 \sin \dfrac{2x}{9}, & \text{when } \dfrac{3\pi}{4} < x < \pi \end{cases}$ then   **[IIT 1991]**

(a) $f(x)$ is continuous at $x = 0$ (b) $f(x)$ is continuous at $x = \pi$

(c) $f(x)$ is continuous at $x = \dfrac{3\pi}{4}$ (d) $f(x)$ is discontinuous at $x = \dfrac{3\pi}{4}$

28. If $f(x) = \begin{cases} 1/2 - x, & 0 < x < 1/2 \\ 0, & x = 0 \\ 1/2, & x = 1/2 \\ 3/2 - x, & 1/2 < x < 1 \\ 1, & x = 1 \end{cases}$, then false statement is   **[Rajasthan PET 1984 (Similar to MP PET 1996)]**

(a) $f(x)$ is discontinuous at $x = 0$ (b) $f(x)$ is continuous at $x = \dfrac{1}{2}$

(c) $f(x)$ is discontinuous at $x = 1$ (d) $f(x)$ is continuous at $x = \dfrac{1}{4}$

29. $f(x) = \dfrac{\sqrt{1+px} - \sqrt{1-px}}{x}, -1 \le x < 0 = \dfrac{2x+1}{x-2}, 0 \le x \le 1$ is continuous in the interval $[-1,1]$ then p equals

(a) -1 (b) $-\dfrac{1}{2}$ (c) $\dfrac{1}{2}$ (d) 1

30. The function $f(x) = \begin{cases} x^2/a & , 0 \le x < 1 \\ a & , 1 \le x < \sqrt{2} \\ (2b^2 - 4b)/x^2, & \sqrt{2} \le x < \infty \end{cases}$ is continuous for $0 \le x < \infty,$ then the most suitable values of a and b

are

(a) $a=1, b=-1$ (b) $a=-1, b=1+\sqrt{2}$ (c) $a=-1, b=1$ (d) None of these

31. Let $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x-2)^2}, & \text{if } x \ne 2 \\ k & , \text{If } x = 2 \end{cases}$ if $f(x)$ be continuous for all x, then $k =$ [IIT 1981]

(a) 7 (b) -7 (c) ± 7 (d) None of these

32. If $f(x) = \begin{cases} \dfrac{x^2 + 3x - 10}{x^2 + 2x - 15}, & \text{when } x \ne -5 \\ a & , \text{when } x = -5 \end{cases}$ is continuous at $x = -5$, then the value of 'a' will be

(a) $\dfrac{3}{2}$ (b) $\dfrac{7}{8}$ (c) $\dfrac{8}{7}$ (d) $\dfrac{2}{3}$

33. The function $f(x) = [x]^2 - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at   [IIT 199

(a) All integers   (b) All integers except 0 and 1   (c) All integers except 0
(d) All integers except 1

34. If $f(x) = \dfrac{1}{2}x - 1$, then on the interval $[0, \pi]$

(a) $\tan[f(x)]$ and $\dfrac{1}{f(x)}$ are both continuous   (b) $\tan[f(x)]$ and $\dfrac{1}{f(x)}$ are both discontinuous

(c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous   (d) $\tan[f(x)]$ is continuous but $\dfrac{1}{f(x)}$ is not continuous

35. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,
(a) $f(x)$ is continuous on R^+ (b) $f(x)$ is continuous on R
(c) $f(x)$ is continuous on $R - Z$ (d) None of these

36. Let $f(x) = [2x^3 - 5]$, $[.]$ denotes the greatest integer function. Then number of points in (1, 2) where the function is discontinuous, is
(a) 0 (b) 13 (c) 10 (d) 3

37. The number of points at which the function $f(x) = \dfrac{1}{x - [x]}$ $[.]$ denotes, the greatest integer function) is not continuous is
(a) 1 (b) 2 (c) 3 (d) None of these

38. If $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{when } x \ne 0 \\ 2 & , \text{when } x = 0 \end{cases}$, then

(a) $\lim\limits_{x \to 0^+} f(x) \ne 2$ (b) $\lim\limits_{x \to 0^-} f(x) = 0$ (c) $f(x)$ is continuous at $x = 0$ (d) None of these

39. The number of points at which the function $f(x) = \dfrac{1}{\log|x|}$ is discontinuous is

(a) 1 (b) 2 (c) 3 (d) 4

40. The function $f(x) = p[x+1] + q[x-1]$, where $[x]$ is the greatest integer function is continuous at $x = 1$ if
(a) $p - q = 0$ (b) $p + q = 0$ (c) $p = 0$ (d) $q = 0$

41. Function $f(x) = \left(1 + \dfrac{x}{a}\right)^{1/x}$ is continuous at $x = 0$, if $f(0)$ equals

(a) e^a (b) e^{-a} (c) 0   (d) $e^{1/a}$

42. Let [.] denote the greatest integer function and $f(x) = [\tan^2 x]$. Then   **[IIT 1993]**

(a) $\lim\limits_{x \to 0} f(x)$ does not exist   (b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$ (d) $f'(0) = 1$

43. The function $f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \pi/4 \\ 2x\cot x + b & \pi/4 \le x \le \pi/2 \\ a\cos 2x - b\sin x & \pi/2 < x \le \pi \end{cases}$ is continuous for $0 \le x \le \pi$ then a, b are

(a) $\dfrac{\pi}{6}, \dfrac{\pi}{12}$ (b) $\dfrac{\pi}{3}, \dfrac{\pi}{6}$ (c) $\dfrac{\pi}{6}, -\dfrac{\pi}{12}$ (d) None of these

44. Let $f : R \to R$ be any function. Define $g : R \to R$ by $g(x) = |f(x)|$ for all x, Then g is   **[IIT 2000]**

(a) Onto if f is onto   (b) One-one if f is one-one

(c) Continuous if f is continuous   (d)   Differentiable if f is differentiable

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Answer Sheet

Assignment (Basic & Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	a	c	b	b	a,b	b	c	b	a	d	c	c	a	c	a	c	d	d	a

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	c	b	c	c	c	c	b	b	c	a	b	d	b	b	b	d	c	c	b

41	42	43	44
d	b	c	c